Using AHP in QFD – The Impact of the ISO 16355 Standard

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- 1981: Dr. Math. ETHZ
- 1991: Six Sigma for Software Black Belt
- 1999: Euro Project Office AG, Zürich
- 2001: Akao Price 2001 for original contributions to QFD
- 2003: SwissICT Expert for Software Metrics
- 2004: Member of the Board QFD Institute Deutschland – QFD Architect
- 2007: CMMI for Software – Level 4 & 5
- 2011: Net Promoter® Certified Associate
- 2012: Member of the DASMA Board
- 2013: Vice-President ISBSG
### Cause-Effect Diagram for Tire Design (Mizuno & Akao, 1994)

<table>
<thead>
<tr>
<th>Process assurance items</th>
<th>Product assurance items (general specifications: items to be assured by plant)</th>
<th>Product assurance items (product specifications)</th>
<th>Assurance items for market</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tread extrusion</td>
<td>Fabrication weight</td>
<td>Feeling of vibrations when driving</td>
<td></td>
</tr>
<tr>
<td>Cutting</td>
<td></td>
<td>Traction &amp; braking capabilities</td>
<td></td>
</tr>
<tr>
<td>Band</td>
<td></td>
<td>Maneuverability</td>
<td></td>
</tr>
<tr>
<td>Bead</td>
<td></td>
<td>Rolling resistance</td>
<td></td>
</tr>
<tr>
<td>Fabrication</td>
<td></td>
<td>External Identifications</td>
<td></td>
</tr>
<tr>
<td>Molding</td>
<td></td>
<td>A</td>
<td></td>
</tr>
<tr>
<td>Inspection</td>
<td></td>
<td>B</td>
<td></td>
</tr>
<tr>
<td>Tread weight</td>
<td>Fabrication weight</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Band weight</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Insulation bead weight</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Squeeze cord unit weight</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mold appearance</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fabrication weight</td>
<td>Product weight</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Product weight</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
First Example of QFD matrix (Suzuki, 1972)

- Impact of Control $x_i$ on Quality $y_j$
- Manual calculation
How Cause-Effect Diagrams become QFD Matrices

\[
\{x_1, x_2, \ldots, x_n\} \rightarrow y \mid x_1, x_2, \ldots, x_n, y \in \mathcal{L}
\]
Comprehensive QFD (Akao, 1990)

- Including quality, technology, cost, and reliability deployments

- House of Quality
- 4-Phase QFD for part suppliers
- Quality Deployment
- New Technology Deployment
- Cost Deployment
- Reliability Deployment
- Customer Deployment
- Function Deployment
- New Concept Deployment
- Components Deployment
- Manufacturing Deployment
- Production Deployment
- Process Improvement
Profiles for Ratio Scales according ISO 16355

- Let \( \mathbf{y} = (y_1, \ldots, y_m) \) be a vector of dimension \( m \). The Euclidian norm for vectors is:

\[
\|\mathbf{y}\| = \sqrt{\sum_{j=1}^{m} y_j^2}
\]

- A vector becomes a **Profile** by dividing components through its length (normalization):

\[
\mathbf{y}' = \frac{\mathbf{y}}{\|\mathbf{y}\|} = \left\{ \frac{y_1}{\|\mathbf{y}\|}, \ldots, \frac{y_m}{\|\mathbf{y}\|} \right\}
\]

- Profiles are vectors of length = 1

- Profiles can be added, subtracted and compared as any other vector
  - Sum of vectors become profiles again be normalization
  - Profiles allow for statistical methods – they show **Directions** in some event space
Calculates weights (sum = 100%)
Calculates profile (sum of squares = 1)
Calculates ranking (for both the same)

The profile is used for the hierarchy because you can compare, add and linearly combine vectors

For weights, this remains forbidden!

The profile is calculated as an Eigenvector, similar to Google Search

The method is shown above

The Annihilator method
Annihilates the matrix $A$ by its transform $A^\top$
Result is $\tau_y$
$AA^\top \tau_y = \tau_y$
A Sample AHP – Which School to Select?

AHP Priorities

### A Education
- **A01 Learning**
  - Weight: 1/4
  - Profile: 0.97
  - Ranking: 1
- **A02 Professions**
  - Weight: 1
  - Profile: 0.24
  - Ranking: 2
- **A03 University Readiness**
  - Weight: 1
  - Profile: 0.97
  - Ranking: 1
- **A04 Musical Education**
  - Weight: 3
  - Profile: 0.24
  - Ranking: 2

### B Social Aspects
- **B01 Friends**
  - Weight: 2
  - Profile: 0.89
  - Ranking: 1
- **B02 Campus Life**
  - Weight: 1
  - Profile: 0.45
  - Ranking: 2

Top Targets

#### Best School
- **A Education**
  - A01 Learning: Ease of learning
  - A02 Professions: Wide range of professions
  - A03 University Readiness: Will successfully conduct studies
  - A04 Musical Education: Learns to perform
- **B Social Aspects**
  - B01 Friends: Makes friends for life
  - B02 Campus Life: Socialize

Profile

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Training</th>
<th>Approach</th>
<th>Behavior</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ease of learning</td>
<td>Trained to learn</td>
<td>Approach</td>
<td>Behavior</td>
</tr>
<tr>
<td>Wide range of professions</td>
<td>Find an excellent start</td>
<td>Approach</td>
<td>Behavior</td>
</tr>
<tr>
<td>Will successfully conduct studies</td>
<td>Well prepared</td>
<td>Approach</td>
<td>Behavior</td>
</tr>
<tr>
<td>Learns to perform</td>
<td>High precision</td>
<td>Approach</td>
<td>Behavior</td>
</tr>
<tr>
<td>Makes friends for life</td>
<td>Good insider relationship</td>
<td>Approach</td>
<td>Behavior</td>
</tr>
<tr>
<td>Socialize</td>
<td>Behavior</td>
<td>Approach</td>
<td>Behavior</td>
</tr>
</tbody>
</table>
Solving a QFD Matrix $y = Ax$
The Trick how to Use AHP Calculation in QFD

\[ y = Ax \]

**Theory**

\[ x_E = A^T y_E \]

**Theory**

\[ x_E = A^T y_E \]

**Eigenvectors:**

<table>
<thead>
<tr>
<th></th>
<th>0.71</th>
<th>-0.69</th>
<th>-0.17</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.59</td>
<td>0.71</td>
<td>-0.39</td>
<td></td>
</tr>
<tr>
<td>0.39</td>
<td>0.17</td>
<td>0.90</td>
<td></td>
</tr>
</tbody>
</table>

**Matrix A**

\[
\begin{bmatrix}
9 & 0 & 2 & 0 \\
0 & 7 & 0 & 5 \\
1 & 2 & 3 & 3 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
9 & 0 & 1 \\
0 & 7 & 2 \\
2 & 0 & 3 \\
0 & 5 & 3 \\
\end{bmatrix}
\]

**Matrix \( A^T A \)**

\[
\begin{bmatrix}
85 & 0 & 15 \\
0 & 74 & 29 \\
15 & 29 & 23 \\
\end{bmatrix}
\]
Advantages of Eigensolution Method

- Eigensolutions are stable
  - When repeatedly applying the process represented by the transfer function $A$, the response $y$ remains always the same
  - $y = AA^T y = AA^T(AA^T y) = AA^T(AA^T(AA^T y)) = \ldots$

- Other solutions might also yield good – even better – convergence gaps but when repeated the process diverges

- Eigensolutions level out inconsistencies
A Measure for Quality – the Convergence Gap

- The **Convergence Gap**

\[ ||y - \tau_y|| = \sqrt{\sum_{i=1}^{m} (y_i - \tau_y_i)^2} \]

reveals the quality of the goal profile’s approximation by the achieved solution profile

- This is the Euclidean Norm
  - Distance between vectors \( y \) and \( \tau_y \)
Displaying QFD Relationship Weights with AHP Judgments

- When using symbols, ISO 16355 proposes cloud symbols
  - $W$ Weak (1)
  - $M$ Medium (3)
  - $S$ Strong (5)
  - $V$ Very Strong (7)
  - $X$ eXtremely strong (9)

<table>
<thead>
<tr>
<th>Importance Level</th>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overruling</td>
<td>$W$</td>
<td>Weak</td>
</tr>
<tr>
<td>8</td>
<td>$W-M$</td>
<td>Somewhat</td>
</tr>
<tr>
<td>7</td>
<td>$M$</td>
<td>Medium</td>
</tr>
<tr>
<td>6</td>
<td>$M-S$</td>
<td>Somewhat</td>
</tr>
<tr>
<td>5</td>
<td>$S$</td>
<td>Strong</td>
</tr>
<tr>
<td>4</td>
<td>$S-V$</td>
<td>Clearly</td>
</tr>
<tr>
<td>3</td>
<td>$V$</td>
<td>Very Strong</td>
</tr>
<tr>
<td>2</td>
<td>$V-X$</td>
<td>Extremely</td>
</tr>
<tr>
<td>1</td>
<td>$X$</td>
<td>Weak</td>
</tr>
<tr>
<td>1/2</td>
<td></td>
<td>Equal</td>
</tr>
<tr>
<td>1/3</td>
<td></td>
<td>Slightly</td>
</tr>
<tr>
<td>1/4</td>
<td></td>
<td>Clearly</td>
</tr>
<tr>
<td>1/5</td>
<td></td>
<td>Very</td>
</tr>
<tr>
<td>1/6</td>
<td></td>
<td>Much</td>
</tr>
<tr>
<td>1/7</td>
<td></td>
<td>No importance</td>
</tr>
<tr>
<td>1/8</td>
<td></td>
<td>Slightly</td>
</tr>
<tr>
<td>1/9</td>
<td></td>
<td>No importance</td>
</tr>
</tbody>
</table>

- Overruling importance
- Much higher importance
- Clearly higher importance
- Somewhat higher importance
- Equal importance
- Somewhat smaller importance
- Clearly smaller importance
- Much smaller importance
- No importance at all
Comprehensive QFD with AHP

AHP for CN

Customer's Needs (CN)

CtQ → CN

Critical to Quality (CtQ)

USt → CN

User Stories (USt)
The ISO 16355 standard projects QFD into the 21\textsuperscript{st} century

QFD is thanks to good mathematics implementable in quality processes

\begin{itemize}
  \item Implement New Lanchester Theory into the New Feature Prioritization concept
  \item Use QFD in agile software development for testing and safety deployment
\end{itemize}

QFD will always depend on the teams using it

\begin{itemize}
  \item Because it record and documents the reasons for taking decisions
\end{itemize}

QFD will go mainstream
Eigensolution level
Inconsistencies out

Similar to Saaty’s
AHP Calculation

Customer's Needs

<table>
<thead>
<tr>
<th>y1 Competency to answer inquiries</th>
<th>0.46</th>
<th>9</th>
<th>9</th>
<th>9</th>
<th>3</th>
<th>3</th>
<th>3</th>
<th>3</th>
<th>9</th>
<th>0.46</th>
</tr>
</thead>
<tbody>
<tr>
<td>y2 Confidentiality</td>
<td>0.35</td>
<td>9</td>
<td>3</td>
<td>9</td>
<td>9</td>
<td>3</td>
<td>9</td>
<td>3</td>
<td>9</td>
<td>0.41</td>
</tr>
<tr>
<td>y3 Suitability for business needs</td>
<td>0.38</td>
<td>3</td>
<td>3</td>
<td>9</td>
<td>9</td>
<td>3</td>
<td>9</td>
<td>3</td>
<td>9</td>
<td>0.40</td>
</tr>
<tr>
<td>y4 Short Development Cycles</td>
<td>0.36</td>
<td>3</td>
<td>3</td>
<td>9</td>
<td>9</td>
<td>3</td>
<td>9</td>
<td>3</td>
<td>9</td>
<td>0.34</td>
</tr>
<tr>
<td>y5 Functionality where you need it</td>
<td>0.34</td>
<td>9</td>
<td>9</td>
<td>1</td>
<td>3</td>
<td>9</td>
<td>9</td>
<td>3</td>
<td>9</td>
<td>0.32</td>
</tr>
<tr>
<td>y6 Social competency</td>
<td>0.35</td>
<td>9</td>
<td>3</td>
<td>9</td>
<td>1</td>
<td>3</td>
<td>9</td>
<td>9</td>
<td>3</td>
<td>0.35</td>
</tr>
<tr>
<td>y7 Communication</td>
<td>0.39</td>
<td>9</td>
<td>3</td>
<td>9</td>
<td>3</td>
<td>9</td>
<td>1</td>
<td>3</td>
<td>9</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Solution Profile for Critical To Quality

0.10 Convergence Range
0.20 Convergence Limit

0.08 Convergence Gap

Critical To Quality
Deployment Combinator

Goal Profile

x1 Browser Style GUI
x2 Keep to XSQL Standard
x3 Open Interfaces
x4 Agile Programming
x5 Reusable Classes
x6 Custom Extensions
x7 Portfolio Management
x8 Reliable Functionality
x9 Moderated Forum

Achieved Profile

0.34
0.30
0.26
0.39
0.25
0.34
0.32
0.32
0.44

Eigensolution level
Inconsistencies out

Similar to Saaty’s
AHP Calculation
• In the columns, two priority profiles are summed up yielding the sum of profiles in the third row, and normalized again in the fourth row.

• Left are the corresponding weight vectors.

• Summing up the weight vectors and transform them back to profiles yields different results than the sum of profiles.

• Summing up the corresponding weight vectors is bad mathematics.
  • Good mathematics is with profiles only.
  • When calculating with weights, large vector components leave a bias.

<table>
<thead>
<tr>
<th>Weights</th>
<th>Profiles</th>
<th>Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>Topic 1</td>
<td>5%</td>
<td>0.00</td>
</tr>
<tr>
<td>Topic 2</td>
<td>85%</td>
<td>0.72</td>
</tr>
<tr>
<td>Topic 3</td>
<td>10%</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>100%</td>
<td>0.86</td>
</tr>
</tbody>
</table>

Weight & Profile 1

<table>
<thead>
<tr>
<th>Weight &amp; Profile 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Topic 1</td>
</tr>
<tr>
<td>Topic 2</td>
</tr>
<tr>
<td>Topic 3</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

Sum of Profiles 1+2

<table>
<thead>
<tr>
<th>Sum of Weights 1+2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Topic 1</td>
</tr>
<tr>
<td>Topic 2</td>
</tr>
<tr>
<td>Topic 3</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

Convergence Gap

\[ 0.24 \]